**The Simple Pendulum (M9)**

**ABSTRACT:** This experiment measured the Earth’s gravitational acceleration (g) by using a simple pendulum with a “massless” string and a point mass. It was also further investigated how the period of a simple pendulum depends on its amplitude. It was measured that g = 9.772 ± 0.002 m/s^2, in almost agreement with the usually accepted value of 9.8 m/s^2.

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**INTRODUCTION**:

A simple pendulum is no more than just a point mass (it is also called the pendulum bob) suspended from a “massless” string. The pendulum bob’s motion can be described as periodic motion, if the pendulum is set swinging in a vertical plane. There are several parameters that can be altered: the mass of the bob (m), the amplitude of the oscillation and the length of the string (L). The period of a pendulum can be expressed mathematically by:

One can vary the pendulum bob’s mass too, however the period theoretically does not depend on mass at all, which is also not investigated in this experiment.

When the amplitudes are small, the period of a pendulum is reduced to this following expression:

This equation still holds reasonable accuracy. In this experiment, the g value is measured using the dependency of the motion on the amplitude of the oscillation.

**PROCEDURE**:

Using vernier calipers, the length and diameter of the brass pendulum bob are measured. The first thumbscrew is tightened to pinch the string between the flat plate on the pendulum clamp. When bob comes to rest, the wing must be adjusted so that 90 degree line is in line with the string. Next, the length L of the pendulum is measured. The total length L is the distance from the center of mass of the bob to the pendulum clamp. It can be measured in two steps. First, measuring the distance from the top of the bob to the pendulum clamp and then adding to this the distance from the top of the bob to its center of mass.

To investigate the pendulum, a controlled experiment; that is, we need to make measurements, changing only one variable at a time. Conducting controlled experiments is a basic principle of scientific investigation. The first step of this experiment investigates the period of a pendulum as a function of length. To do this, the photogate (device that allows for extremely accurate timing of events within physics experiments) must be arranged so that its beam is blocked through the oscillation of the pendulum. The computer software connected to the photogate measures the time for a complete oscillation. This period is measured for five different values of L in the range 0 to 100 cm, keeping everything else the same including the amplitude (angle). The angle must be fifteen degrees or less. For each given value this is performed three times.

The following raw data table is generated:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| type | trial # | length (cm) | angle (°) | T(s) |
| 1 | 1 | 61,86 | 10,00 | 1,57 |
| 2 | 61,86 | 10,00 | 1,57 |
| 3 | 61,86 | 10,00 | 1,57 |
| 2 | 1 | 54,96 | 10,00 | 1,48 |
| 2 | 54,96 | 10,00 | 1,48 |
| 3 | 54,96 | 10,00 | 1,48 |
| 3 | 1 | 51,96 | 10,00 | 1,44 |
| 2 | 51,96 | 10,00 | 1,44 |
| 3 | 51,96 | 10,00 | 1,44 |
| 4 | 1 | 48,96 | 10,00 | 1,39 |
| 2 | 48,96 | 10,00 | 1,39 |
| 3 | 48,96 | 10,00 | 1,39 |
| 5 | 1 | 35,46 | 10,00 | 1,19 |
| 2 | 35,46 | 10,00 | 1,19 |
| 3 | 35,46 | 10,00 | 1,19 |

As discussed earlier, as long as the amplitudes are small, this expression is valid for the period:

After the first step is completed, a graph of T^2 vs. L drawn and the value of g implied by data is determined (results and analysis).

|  |
| --- |
| T^2 (s^2) |
| 2,47 |
| 2,48 |
| 2,47 |
| 2,18 |
| 2,18 |
| 2,18 |
| 2,07 |
| 2,07 |
| 2,07 |
| 1,94 |
| 1,94 |
| 1,94 |
| 1,40 |
| 1,40 |
| 1,40 |

For the second part of the experiment, the length must be held constant while the angle (amplitude) is changed. To do this, L is fixed in the range 50 to 100 cm. The period as a function of amplitude is measured over the range 5 degrees to 45 degrees, in 5 steps. The raw data table is generated after collecting data with photogate.

|  |  |  |  |
| --- | --- | --- | --- |
| trial # | length (cm) | angle (°) | T(s) |
| 1 | 61,96 | 5,00 | 1,57 |
| 2 | 61,96 | 10,00 | 1,58 |
| 3 | 61,96 | 15,00 | 1,58 |
| 4 | 61,96 | 20,00 | 1,58 |
| 5 | 61,96 | 25,00 | 1,59 |
| 6 | 61,96 | 30,00 | 1,59 |
| 7 | 61,96 | 35,00 | 1,60 |
| 8 | 61,96 | 40,00 | 1,61 |
| 9 | 61,96 | 45,00 | 1,62 |

Since the amplitudes are large in this trial, the expression below is valid,

which can also be written as:

Where

The last step of the procedure is to use Regression Analysis to see how well the data fits that expression. To do that, four columns are created : amplitude (in radians) x1, x2 and period. The amplitude and period values are directly taken by the data collected from the experiment. Excel will calculate the X1 and X2. To get a chart summarizing the regression, Excel’s regression analysis feature is used.

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| SUMMARY OUTPUT | | |  | | |  | |  | |  |  |
|  | | |  | | |  | |  | |  |  |
| *Regression Statistics* | | | | | |  | |  | |  |  |
| Multiple R | | | 0,999037421 | | |  | |  | |  |  |
| R Square | | | 0,998075769 | | |  | |  | |  |  |
| Adjusted R Square | | | 0,997434359 | | |  | |  | |  |  |
| Standard Error | | | 0,000726309 | | |  | |  | |  |  |
| Observations | | | 9 | | |  | |  | |  |  |
|  |  | | |
| ANOVA | | |  | | |  | |  | |  |  |
|  | | | *df* | | | *SS* | | *MS* | | *F* | *Significance F* |
| Regression | | | 2 | | | 0,001641724 | | 0,000820862 | | 1556,064742 | 7,12478E-09 |
| Residual | | | 6 | | | 3,16515E-06 | | 5,27524E-07 | |  |  |
| Total | | | 8 | | | 0,001644889 | |  | |  |  |
|  | | |  | | |  | |  | |  |  |
|  | | | *Coefficients* | | | *Standard Error* | | *t Stat* | | *P-value* | *Lower 95%* |
| Intercept | | | 1,573492098 | | | 0,000499243 | | 3151,757533 | | 6,88631E-20 | 1,572270495 |
| X Variable 1 | | | 0,268960347 | | | 0,018325215 | | 14,67706378 | | 6,28259E-06 | 0,224120161 |
| X Variable 2 | | | 0,096094422 | | | 0,124873731 | | 0,769532719 | | 0,470776994 | -0,209460591 |
|  |  | | |
|  | |  | | |  | |  | |

The values in first two colmns of the bottom table refer to a, ab and ac. The pure constant values of a,b, and c and their respective uncertainties are to be found (analysis).

**RESULTS AND ANALYSIS:**

This is a graph of T^2 vs. L for small amplitudes when this equation is valid:

This is a function for period with respect to length. We must rearrange this equation to read the slope from the graph accurately.

The uncertainty is therefore

To show our fit values for a, b, and c we use the following formula.

Hence, the experimental values are:

As opposed the expected values:

**CONCLUSION**:

All of the figures above match the expected values except for b, which is slightly off the calculated value. This discrepancy is due to experimental errors such as those caused by friction, unvertical releases of the bob and the uncertain exact amplitudes.

This formula above gives us: 1.579 s for the conducted experiment. However, as the experimental values suggest, it increases up to 1.62 s with larger amplitudes. For the angle 15 degrees we multiply that 1.579s with 1.0001 which introduces 0.00157 error in the actual value. This amount of error can be overlooked. Anything above 15 degrees does not count as a “small amplitude” anymore because of the larger error (greater than 0.1%) introduced in the calculations, so we cannot use the simplified formula.